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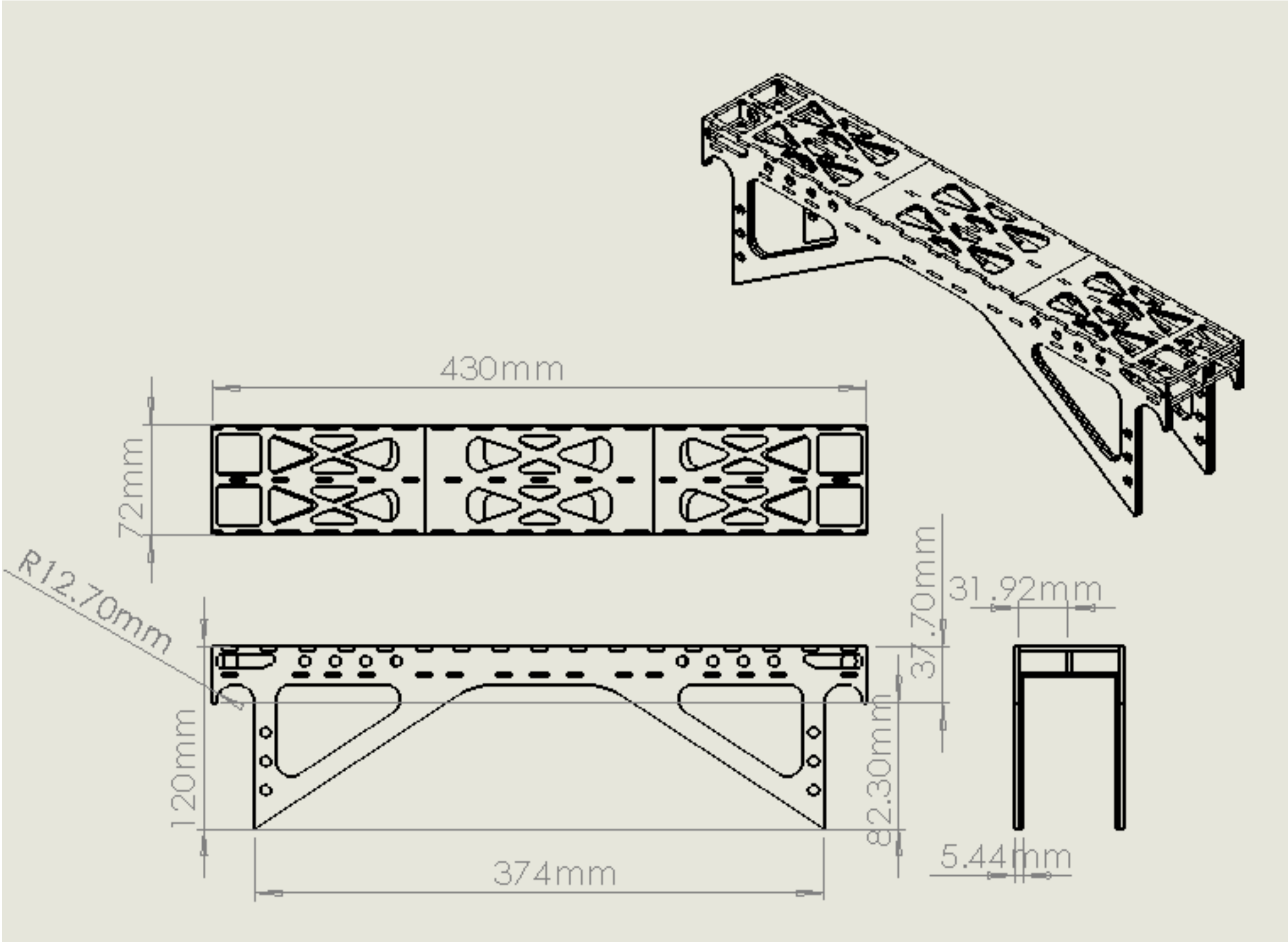
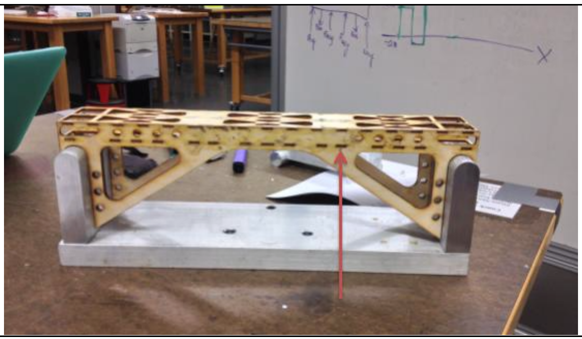
Group 1
ENGS 033 Final Bridge Report

Final Tested Design:



SolidWorks

Predicted Results vs. Actual Results:

	Deflection under 1kN	Critical Load	Location of Failure
Predicted Results	0.1 mm	1.7 kN	
Actual Results	1.678 mm	3.384 kN	Buckling of side truss members 

Analytical Results: See attached.

Post-testing Discussion:

During testing, the bridge failed when the angled side truss members buckled under the critical load. It failed there because the strength of the I-beam deck combined with the support from the jig on the vertical members of the side trusses left the angled truss members more vulnerable to buckling.

Our predicted deflection under 1 kN of force was approximately 6% of the actual deflection under 1 kN. Also, our critical load prediction was approximately 50.2% of the actual critical load applied before failure. However, our bridge did relatively well concerning the failure load to weight ratio and bridge size: The failure to weight ratio was approximately 17.66 N/gram, the highest ratio of all the groups, and the bridge weight was about 191.6 grams, the smallest of all the groups. This was beneficial for our bridge because optimal values for these two characteristics are critical in bridge construction and material management. In addition, our bridge performed relatively well in the deflection*weight category with 321.5 mm*grams.

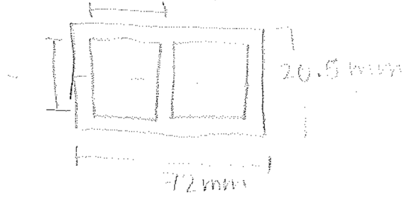
To improve the accuracy of the analysis, we could have been less conservative with our Phase 3 calculations. We originally had a predicted value of about 3.0 kN and also had a SolidWorks predicted value of 3.6 kN as the critical load; however, we decided to take into consideration imperfection factors such as gluing and bridge assembly, causing us to decrease our predicted value by about a factor of 2 to 1.7 kN. Without consideration of those factors, our load prediction would have been more accurate at about 88.7% of the actual critical load. The deflection prediction could have been improved by considering that SolidWorks does not take buckling into account significantly during model analysis and that SolidWorks also takes into account warping of the model that is realistically unfeasible when calculating deflection. Realizing this, we could have increased our prediction value for deflection.

In order to improve the scale-bridge model, we could increase the strength in the side trusses that had buckled during testing by adding a cross-brace between adjacent trusses on the shorter sides of the assembly in order to limit buckling. We could also reinforce the deck with a smaller upper truss or a more elaborate lower truss to limit buckling in the lower truss region.

Analytical Calculations

Note About predictions ⓪

Deck simplified as box beam



$I = \frac{1}{12}(0.072 \times 0.0205 - 2 \times \quad \times \quad)$

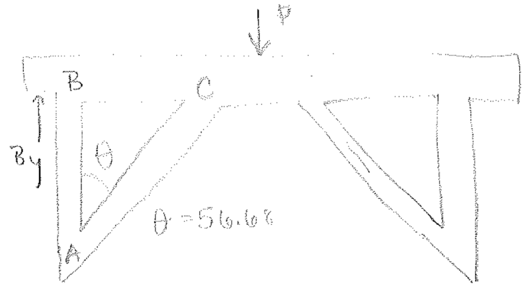
$\approx 3.4(10^{-8})\, m^4$

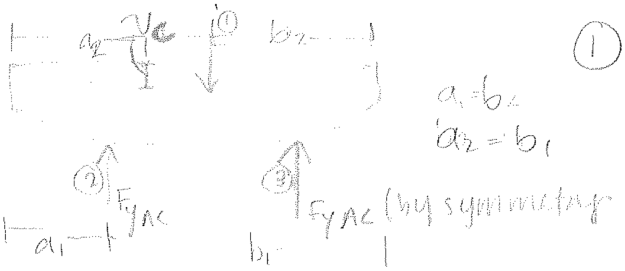
70 MPa → compression 70% as good → 50 MPa

lowest value 3.0 kN

↳ to account for bridge imperfections → use factor of 2

$\frac{3.0}{2} =$ 17 kN is the critical load



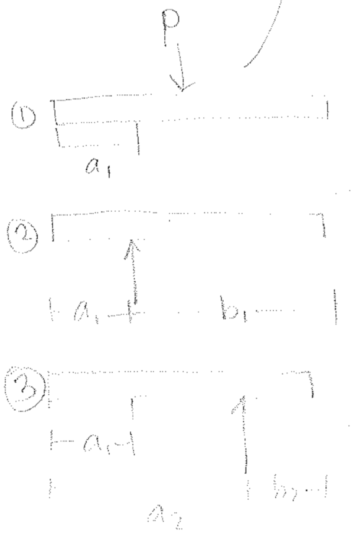


$a_1 = b_2$
$a_2 = b_1$

POINT FORCE @ center

$V_c = \delta_{AC}$ due to F_{AC} developed

$$\frac{F_{AC}L}{A\cancel{E}} = \overset{①}{\frac{-Pa}{48\cancel{E}I}(3L^2-4a^2)} + \overset{②}{\frac{F_{AC}\cos(56.68)\,b_1a_1(L^2-b_1^2-a_1^2)}{6\cancel{E}IL}} + \frac{F_{AC}\cos(56.68)\,b_2a_1(L^2-b_2^2-a_1^2)}{6\cancel{E}IL}③$$



$$48F_{AC}L^2I = -PAa_1L(3L^2-4a^2) + A8F_{AC}\cos(56.68)\,a_1\left[b_1(L^2-b_1^2-a_1^2)+b_2(L^2-b_2^2-a_1^2)\right]$$

$$F_{AC}\left[48L^2I - 8A\cos(56.68)\,a_1[\sim]\right] = -PAa_1L(3L^2-4a^2)$$

$$F_{AC} = \frac{-PAa_1L(3L^2-4a_1^2)}{48L^2I - 8A\cos(56.68)\,a_1\left[b_1(L^2-b_1^2-a_1^2)+b_2(L^2-b_2^2-a_1^2)\right]}$$

$a_1 = .160\text{m}$
$b_1 = .246\text{m}$
$b_2 = .160\text{m}$
$L = .406\text{ m}$
$I = 3.4\times10^{-8}\text{m}^4$
$A = 2\times0.00272\times0.012\text{m}^2$
$= 6.528E{-}5$

$$F_{AC} = \frac{-P(6.528E{-}5)(.16)(.406)\left[3(.406)^2-4(.16)^2\right]}{48(.406^2)(3.4E{-}8)-8(6.528E{-}5)\cos(56.68)(.16)\left[(.246)(.406^2-.246^2-.16^2)+(.16)(.406^2-.16^2-.16^2)\right]}$$

$$F_{AC} = \frac{-P(1.663E{-}6)}{2.69E{-}7-1.735E{-}6} = F_{AC} = .1.7(P$$

RANGE
70 MPa → 2.6 kN
60MPa → 2.3kN
50MPa → 1.8kN
45MPa → 1.7kN

$F_{AC} = .702P$ → compression

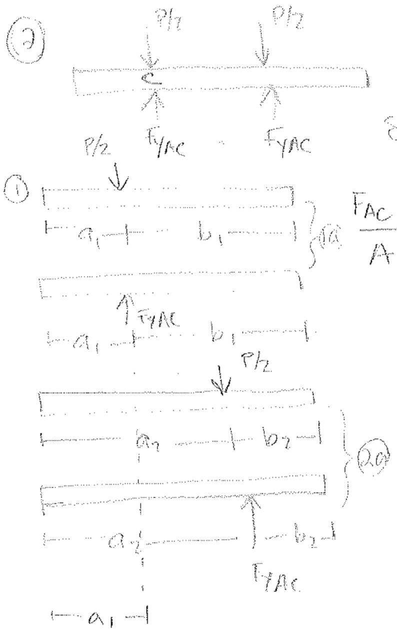
1/8" cross grain in tension, $\sigma_{max} \sim 70$ MPa
wood performs worse in compression

→ let's use $\sigma_{max} \sim 45$ MPa (60%)

$$45\text{ MPa} = \frac{1.7P}{A}$$

$$\frac{45(10^6)(6.52E{-}5)}{1.7} = 1.7\text{ kN}$$

2 LOADS 150 mm APART



$\delta_{AC} = V_C$

(1a)

$$\frac{F_{AC}L}{A\cancel{E}} = \frac{\left(F_{AC}\cos(56.68) - \frac{P}{2}\right)\overbrace{(b_1a_1)(L_2 - b_1^2 - a_1^2)}^{[A]}}{6\cancel{E}IL} + \frac{\left(F_{AC}\cos(56.68) - \frac{P}{2}\right)\underbrace{(b_2a_1)(L_2 - b_2^2 - a_1^2)}_{[B]}}{6\cancel{E}IL}$$

$$6F_A L^2 I = AF_{AC}\cos(56.68)([A]+[B]) - A\frac{P}{2}([A]+[B])$$

$$F_{AC}\left(6L^2I - A\cos(56.68)([A]+[B])\right) = -A\frac{P}{2}([A]+[B])$$

$$F_{AC} = \frac{-AP([A]+[B])}{6L^2I - A\cos(56.68)([A]+[B])}$$

$a_1 = .160$ m
$b_1 = .246$ m
$b_2 = .160$ m
$L = .406$ m
$I = 3.4(10^{-8})\,m^4$
$A = 6.528(10^{-5})\,m^2$

$[A] = (.246)(.160)(.406^2 - .246^2 - .160^2) = 0.0030984$
$[B] = (.160)(.160)(.406^2 - .160^2 - .160^2) = 0.00290908$

$$F_{AC} = \frac{-P(3.92E-7)}{3.36E-8 - 2.145E-7} = 2.17P$$

$$\sigma_{max} = \frac{2.17P}{6.528(10^{-5})}$$

σmax	Pmax
70 MPa	2.1 kN
60 MPa	1.8 kN
50 MPa	1.5 kN
40 MPa	1.2 kN

used 1/8" data because ply structure for  is different from 
1/8+1/8 — 1/4
but, with the glue present, it's probably between the two (in tension)

actually, load is distributed over 2 trusses

σmax(MPa)	Pmax(kN)
70	4.2
60	3.6
50	3.0
40	2.4

— solidworks material value

Predictions

Deflection with 1kN
0.1 mm

Critical Load
with imperfections (1.7 kN)
raw 3.0 kN → 3.6 kN

③

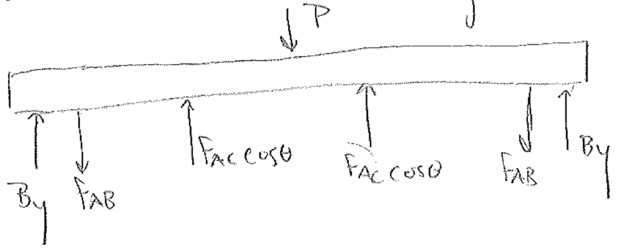
Using the value for F_{AC} found on page 1, a point analysis of A reveals that $F_{AB} = F_{AC} \cos(56.68)$ $F_{AC} = 1.7P$

$\theta = 56.68°$

$\sum F_y = 0 = F_{AB} - F_{AC}\cos(\theta)$ $F_{AB} = .93P$

A quick analysis of external forces indicates that $B_y = \frac{P}{2}$ and that the two sides of the bridge are symmetrical.

Now to consider the bridge deck:



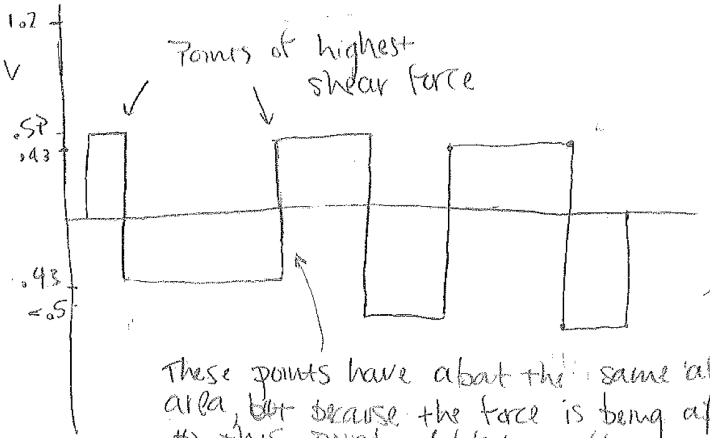
$\tau_{max} = 1.5\frac{V}{A}$ $A = (.0107)(.00544)$

$A = 5.8208 \times 10^{-5}\ m^2$

$\tau_{max} = 12884.83P$

τ_{max} range →	Breaking force
70 Mpa →	5.4 KN
60 Mpa →	4.6 KN
50 Mpa →	3.9 KN
45 Mpa →	3.4 KN

shear force diagram:



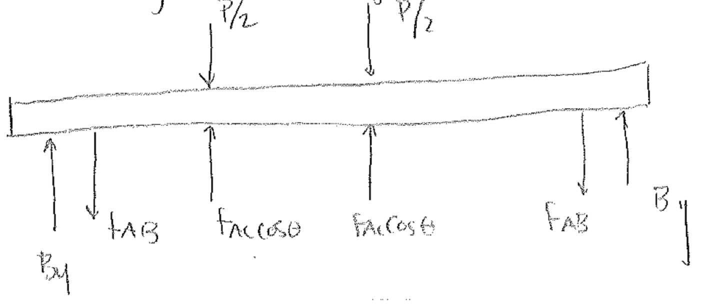
These points have about the same affective area, but because the force is being applied closer to this point deflection affects it more, meaning it's more likely to break there first.

Analysis for point force at Center
* Bridge Deck

Using the Valuo found on page 2, and the point analysis of A on page 3 $F_{AB} = F_{AC}\cos\theta = 1.192P$

External force analysis reveals ~~that~~ By still equals $P/2$

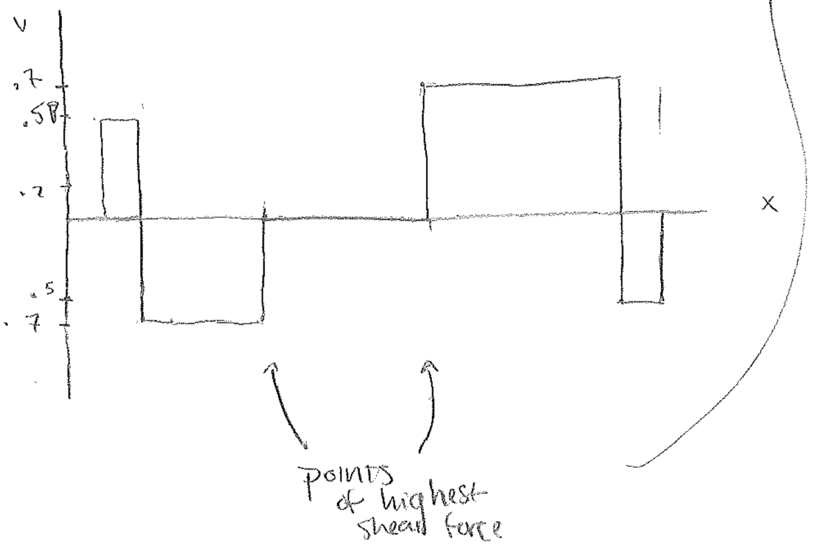
Considering the bridge deck:



$\tau_{max} = 1.5\frac{V}{A}$ $A = 5.8208 \times 10^{5} m$

$\tau_{max} = \frac{(1.5)(.692)}{A}P = 178326 P$

Shear force diagram



τ_{max} range →	Breaking force.
70 Mpa →	3.9 KN
60 Mpa →	3.36 KN
50 Mpa →	2.8 KN
45 Mpa →	2.5 KN

Analysis for two point forces 150 mm apart
* Bridgedeck